Applied Soft Computing 9 (2009) 457–461

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Extension of fuzzy TOPSIS method based on interval-valued fuzzy sets

Behzad Ashtiani [a,*], Farzad Haghighirad [b], Ahmad Makui [a], Golam ali Montazer [b]

[a] Department of Industrial Engineering, Iran University of Science and Technology, Narmak, 16846-13114, Tehran, Iran
[b] Department of Industrial Engineering, Tarbiat Modares University, Iran

ARTICLE INFO

Article history:
Received 20 February 2007
Received in revised form 20 May 2008
Accepted 26 May 2008
Available online 17 July 2008

Keywords:
TOPSIS
Multi criteria decision making
Interval-valued fuzzy set

ABSTRACT

Decision making is one of the most complex administrative processes in management. In circumstances where the members of the decision making team are uncertain in determining and defining the decision making criteria, fuzzy theory provides a proper tool to encounter with such uncertainties. However, if decision makers cannot reach an agreement on the method of defining linguistic variables based on the fuzzy sets, the interval-valued fuzzy set theory can provide a more accurate modeling. In this paper the interval-valued fuzzy TOPSIS method is presented aiming at solving MCDM problems in which the weights of criteria are unequal, using interval-valued fuzzy sets concepts.

1. Introduction

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is a popular approach to multiple criteria decision making (MCDM) problems that was proposed by Hwang and Yoon [1,2]. This method has been widely used in the literature (Abo-sinna and Amer [3], Agrawal et al. [4], Chen and Tzeng [5]). TOPSIS is a multiple criteria method to identify solution from a set of finite alternatives. The basic principle is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. In the TOPSIS, the performance ratings and the weights of the criteria are given as crisp values.

Among many cases, crisp data are inadequate to model real life situations. Chen [6] extends the TOPSIS method to fuzzy group decision making situations by considering triangular fuzzy numbers and defining crisp Euclidean distance between two fuzzy numbers. Tsaur et al. [7] convert fuzzy MCDM problem into a crisp one via centroid defuzzification and then solve the non-fuzzy MCDM problem using the TOPSIS method. Chu and Lin [8] changed fuzzy MCDM problem into a crisp one. Differing from the others, they first derive the membership functions of all the weighted ratings in a weighted normalized decision matrix and then by defuzzifying, convert them to crisp values. Triantaphyllou and Lin [9] develop a fuzzy version of TOPSIS method based on fuzzy arithmetic operations, which leads to a fuzzy relative closeness for each alternative.

Except for Wang and Elhag [10], our survey shows that the vital shortcoming in the other mentioned methods are the loss of information (defuzzification) in initial steps of their procedure. Wang and Elhag's fuzzy TOPSIS method is based on alpha level sets and the fuzzy extension principle, which compute the fuzzy relative closeness of each alternative by solving the Non-linear programming models. Final ranking is obtained by defuzzifying the fuzzy relative closeness values.

In this paper we develop an interval-valued fuzzy TOPSIS (IVF-TOPSIS) to solve MCDM problems in which the performance rating values as well as the weights of criteria are linguistics terms which can be expressed in interval-valued fuzzy (IVFN) numbers. The remaining of this paper is organized as follows. In the next section, we will briefly introduce the TOPSIS method. Section 3 illustrates interval-valued fuzzy sets (IVFS). Section 4 describes developed TOPSIS method to solve interval-valued fuzzy MCDM problems. Section 5 investigates a numerical example including an application to select a manager for R&D department in a company. The paper is concluded in Section 6.

2. TOPSIS method

Yoon and Hwang [1,2], introduced the TOPSIS method based on the idea that the best alternative should have the shortest distance from an ideal solution. They assumed that if each attributes takes monotonically increasing or decreasing variation, then it is easy to define an ideal solution. Such a solution is composed of all the best attributes values achievable, while the worst solution is composed of all worst attribute values achievable.

* Corresponding author. Tel.: +9821 73913000.
E-mail addresses: ashtiani@iust.ac.ir, behzad.ashtiani@econ.kuleuven.be (B. Ashtiani), haghighirad@modares.ac.ir (F. Haghighirad), amakui@iust.ac.ir (A. Makui), montazer@modares.ac.ir (G.a. Montazer).

1568-4946/$ – see front matter
doi:10.1016/j.asoc.2008.05.005

Suppose a multi criteria decision making problem having n alternatives, A_1, A_2, ..., A_n and m criteria, C_1, C_2, ..., C_m. Each alternative is evaluated with respect to the m criteria. All the values/ratings are assigned to alternatives with respect to decision matrix denoted by $X(x_{ij})_{n\times m}$. Let $W = (w_1, w_2, \ldots, w_m)$ be the weight vector of criteria, satisfying $\sum_{j=1}^{m} w_j = 1$.

The TOPSIS method consists of the following steps:

i. Normalize the decision matrix: the normalization of the decision matrix is done using the following transformation for each r_{ij}.

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^{n} x_{kj}^2}}, \quad i = 1, \cdots, n; \quad j = 1, \cdots, m. \tag{1}$$

Multiply the columns of the normalized decision matrix by the associated weights. The weighted and normalized decision matrix is obtained as:

$$V_{ij} = w_j \times r_{ij}; \quad i = 1, \cdots, n \quad j = 1, \cdots, m \tag{2}$$

where w_j represents the weight of the jth criterion.

ii. Determine the ideal and negative ideal alternatives: the ideal and negative ideal alternatives are determined, respectively, as follows:

$$A^* = \{v_1^*, v_2^*, \cdots, v_m^*\} = \left\{\left(\max_j v_{ij} | j \in \Omega_b\right), \left(\min_j v_{ij} | j \in \Omega_c\right)\right\} \tag{3}$$

$$A^- = \{v_1^-, v_2^-, \cdots, v_m^-\} = \left\{\left(\min_j v_{ij} | j \in \Omega_b\right), \left(\max_j v_{ij} | j \in \Omega_c\right)\right\} \tag{4}$$

where Ω_b is the set of benefit criteria and Ω_c is the set of cost criteria.

iii. Obtain the distance of the existing alternatives from ideal and negative ideal alternatives: the two Euclidean distances for each alternative are, respectively, calculated as:

$$S_i^+ = \sqrt{\sum_{j=1}^{m} (v_{ij} - v_j^*)^2}, \quad i = 1, \cdots, n \tag{5}$$

$$S_i^- = \sqrt{\sum_{j=1}^{m} (v_{ij} - v_j^-)^2}, \quad i = 1, \cdots, n. \tag{6}$$

iv. Calculate the relative closeness to the ideal alternatives: the relative closeness to the ideal alternatives can be defined as:

$$RC_i = \frac{S_i^-}{S_i^+ + S_i^-}, \quad i = 1, 2, \cdots, m, \quad 0 \le RC_i \le 1. \tag{7}$$

v. Rank the alternatives: according to the relative closeness to the ideal alternatives, the bigger is the RC_i, the better is the alternative A_i.

3. Interval-valued fuzzy sets

In fuzzy sets theory, it is often difficult for an expert to exactly quantify his or her opinion as a number in interval [0,1]. Therefore, it is more suitable to represent this degree of certainty by an interval. Sambuc [12] and Grattan [13] noted that the presentation of a linguistic expression in the form of fuzzy sets is not enough. Interval-valued fuzzy sets were suggested for the first time by Gorzlczany [14] and Turksen [15]. Also Cornelis et al. [16] and Karnik and Mendel [17] noted that the main reason for proposing this new concept is the fact that in the linguistic modeling of a phenomenon, the presentation of the linguistic expression in the form of ordinary fuzzy sets is not clear enough. Wnag and Li [18] defined interval-valued fuzzy numbers (IVFN) and gave their extended operations. Interval-valued fuzzy sets have been widely used in real-world applications. For instance, Kohout and Bandler [19] in a CLINAID system, Sambuc [12] in thyrodian pathology, Gorzlczany [14] and Bustine [20] in approximate reasoning, Turksen [21,22] in interval-valued logic and in preference modeling [15]. Based on definition of interval-valued fuzzy set in [14], an interval-valued fuzzy set A defined on $(-\infty,+\infty)$ is given by:

$$\begin{aligned} &A = \{(x, [\mu_A^L(x), \mu_A^U(x)]\} \\ &\mu_A^L, \mu_A^U : X \to [0,1] \quad \forall x \in X, \quad \mu_A^L \le \mu_A^U \\ &\bar{\mu}_A(x) = [\mu_A^L(x), \mu_A^U(x)] \\ &A = \{(x, \bar{\mu}_A(x))\}, x \in (-\infty, \infty) \end{aligned} \tag{8}$$

where $\mu_A^L(x)$ is the lower limit of degree of membership and $\mu_A^U(x)$ is the upper limit of degree of membership.

Fig. 1 illustrates the membership value at $x'$of interval-valued fuzzy set A. Thereby, the minimum and maximum membership value of x' are $\mu_A^L(x)$ and $\mu_A^U(x)$, respectively.

Given two interval-valued fuzzy numbers $N_x = [N_x^-; N_x^+]$ and $M_y = [M_y^-; M_y^+]$, according to [23,24], we have:

Definition 1. If $\cdot \in (+,-,\times,\div)$, then $N \cdot M(x \cdot y) = [N_x^- \cdot M_y^-; N_x^+ \cdot M_y^+]$

Definition 2. The Normalized Euclidean distance between $\tilde{N}$ and $\tilde{M}$ is as follows:

$$D(\tilde{N}, \tilde{M}) = \sqrt{\frac{1}{6}\sum_{i=1}^{3}[(N_{x_i}^- - M_{y_i}^-)^2 + (N_{x_i}^+ - M_{y_i}^+)^2]}$$

4. The proposed interval-valued fuzzy TOPSIS

In fuzzy MCDM problems, performance rating values and relative weights are usually characterized by fuzzy numbers. A fuzzy number is a convex fuzzy set, defined by a given interval of real numbers, each with a membership value between 0 and 1. Considering the fact that, in some cases, determining precisely of this value is difficult, the membership value can be expressed as an interval, consisting real numbers. In this paper criteria values' as well as criteria weights', are considered as linguistic variables. The concept of linguistic variable is very useful in dealing with situations that are too complex or ill-defined to be reasonably described in conventional quantitative expressions [11]. These linguistic variables can be converted to triangular interval-valued fuzzy numbers as depicted in Tables 1 and 2.

Let $\tilde{X} = [\tilde{x}_{ij}]_{n\times m}$ be a fuzzy decision matrix for a multi criteria decision making problem in which A_1, A_2, ..., A_n are n possible alternatives and $C_1, C_2, \ldots, C_m$ are m criteria. So the performance of alternative A_i with respect to criterion C_j is denoted as $\tilde{x}_{ij}$. As illustrated in Fig. 2, $\tilde{x}_{ij}$ and $\tilde{w}_j$ are expressed in triangular interval-valued fuzzy numbers.

$$\tilde{x} = \begin{cases} (x_1, x_2, x_3) \\ (x_1', x_2, x_3') \end{cases}$$

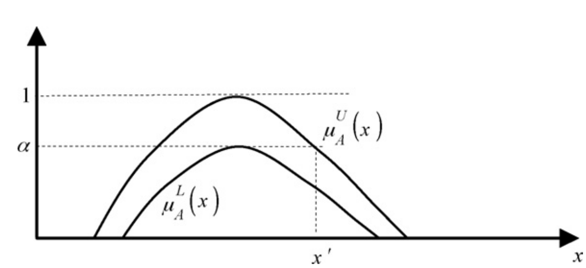

Fig. 1. Interval-valued fuzzy set.

Table 1
Definitions of linguistic variables for the ratings

Very Poor (VP)	[(0,0);0;(1,1.5)]
Poor (P)	[(0,0.5);1;(2.5,3.5)]
Moderately Poor (MP)	[(0,1.5);3;(4.5,5.5)]
Fair (F)	[(2.5,3.5),5,(6.5,7.5)]
Moderately Good (MG)	[(4.5,5.5),7,(8,9.5)]
Good (G)	[(5.5,7.5),9,(9.5,10)]
Very Good (VG)	[(8.5,9.5),10,(10,10)]

Table 2
Definitions of linguistic variables for the importance of each criterion

Very low (VL)	[(0,0);0;(0.1,0.15)]
Low (L)	[(0,0.05);0.1;(0.25,0.35)]
Medium low (ML)	[(0,0.15);0.3;(0.45,0.55)]
Medium (M)	[(0.25,0.35),0.5,(0.65,0.75)]
Medium high (MH)	[(0.45,0.55),0.7,(0.8,0.95)]
High (H)	[(0.55,0.75),0.9,(0.95,1)]
Very high (VH)	[(0.85,0.95),1,(1,1)]

The $\tilde{x}$ can be also demonstrated as $\tilde{x} = [(x_1, x'_1); x_2; (x'_3, x_3)]$. It is worth noting, the use of interval value numbers gives an opportunity to experts to define lower and upper bounds values as an interval for matrix's elements and weights of criteria. Also in a group decision environment with K persons, the importance of the criteria and the rating of alternatives with respect to each criterion can be calculated as:

$$\tilde{x}_{ij} = \frac{1}{K}[\tilde{x}^1_{ij} + \tilde{x}^2_{ij} + \cdots + \tilde{x}^k_{ij}]$$
$$\tilde{w}_{ij} = \frac{1}{K}[\tilde{w}^1_{ij} + \tilde{w}^2_{ij} + \cdots + \tilde{w}^k_{ij}] \tag{9}$$

Eq. (9) represents the average values of x_{ij} and w_{ij} denoted by experts, where (+) is the sum operator and is applied to the interval-valued fuzzy numbers as defined in Definition 1. So the output is also an interval value fuzzy number.

Now the proposed approach to develop the TOPSIS for interval-valued fuzzy data (IVF-TOPSIS) can be defined as follows:

i. Given $\tilde{x}_{ij} = [(a_{ij}, a'_{ij}); b_{ij}; (c'_{ij}, c_{ij})]$, the normalized performance rating as an extension to Chen [6] can be calculated as:

$$\tilde{r}_{ij} = \left[\left(\frac{a_{ij}}{c^+_j}, \frac{a'_{ij}}{c^+_j}\right); \frac{b_{ij}}{c^+_j}; \left(\frac{c'_{ij}}{c^+_j}, \frac{c_{ij}}{c^+_j}\right)\right], \quad i = 1, \ldots, n, \quad j \in \Omega_b$$
$$\tilde{r}_{ij} = \left[\left(\frac{a^-_j}{a'_{ij}}, \frac{a^-_j}{a_{ij}}\right); \frac{a^-_j}{b_{ij}}; \left(\frac{a^-_j}{c_{ij}}, \frac{a^-_j}{c'_{ij}}\right)\right], \quad i = 1, \ldots, n, \quad j \in \Omega_c \tag{10}$$
$$c^+_j = \operatorname{Max}_i c_{ij}, \quad j \in \Omega_b$$
$$a^-_j = \operatorname{Min}_i a'_{ij}, \quad j \in \Omega_c$$

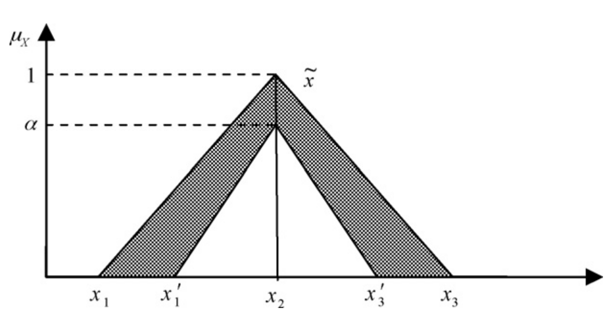

Fig. 2. Interval-valued triangular fuzzy number.

The normalization logic used in (10) is the same with which is used in deterministic problems.

Hence, the normalized matrix $\tilde{R} = [\tilde{r}_{ij}]_{n\times m}$ can be obtained.

ii. By considering the different importance of each criterion, we can construct the weighted normalized fuzzy decision matrix as: $\tilde{V} = [\tilde{v}_{ij}]_{n\times m}$ where $\tilde{v}_{ij} = \tilde{r}_{ij} \times \tilde{w}_j$. According to Defintion 1, the multiply operator can be applied as:

$$\tilde{v}_{ij} = [(\tilde{r}_{1_{ij}} \times \tilde{w}_{1_j}, \tilde{r}'_{1_{ij}} \times \tilde{w}'_{1_j}); \tilde{r}_{2_{ij}} \times \tilde{w}_{2_j}; (\tilde{r}'_{3_{ij}} \times \tilde{w}'_{3_j}, \tilde{r}_{3_{ij}} \times \tilde{w}_{3_j})]$$
$$= [(g_{ij}, g'_{ij}); h_{ij}; (l_{ij}, l'_{ij})] \tag{11}$$

iii. Ideal and negative ideal solution can be defined as:

$$A^+ = [(1,1);1;(1,1)], \quad j \in \Omega_b$$
$$A^- = [(0,0);0;(0,0)], \quad j \in \Omega_c \tag{12}$$

iv. Normalized Euclidean distance can be calculated using Definition 2 as follows:

$$D^-(\tilde{N}, \tilde{M}) = \sqrt{\frac{1}{3}\sum_{i=1}^{3}[(N^-_{x_i} - M^-_{y_i})^2]}$$
$$D^+(\tilde{N}, \tilde{M}) = \sqrt{\frac{1}{3}\sum_{i=1}^{3}[(N^+_{x_i} - M^+_{y_i})^2]}$$

where $D^-(\tilde{N}, \tilde{M})$ and $D^+(\tilde{N}, \tilde{M})$ are the primary and secondary distant measure, respectively. Thereby, distance of each alternative from the ideal alternative $[D^+_{i1}, D^+_{i2}]$ can be currently calculated, where:

$$D^+_{i1} = \sum_{j=1}^{m}\sqrt{\frac{1}{3}[(g_{ij} - 1)^2 + (h_{ij} - 1)^2 + (l_{ij} - 1)^2]}$$
$$D^+_{i2} = \sum_{j=1}^{m}\sqrt{\frac{1}{3}[(g'_{ij} - 1)^2 + (h_{ij} - 1)^2 + (l'_{ij} - 1)^2]} \tag{13}$$

Similarly, the separation from the negative ideal solution is given by $[D^-_{i2}, D^-_{i1}]$, where:

$$D^-_{i1} = \sum_{j=1}^{m}\sqrt{\frac{1}{3}[(g_{ij} - 0)^2 + (h_{ij} - 0)^2 + (l_{ij} - 0)^2]}$$
$$D^-_{i2} = \sum_{j=1}^{m}\sqrt{\frac{1}{3}[(g'_{ij} - 0)^2 + (h_{ij} - 0)^2 + (l'_{ij} - 0)^2]} \tag{14}$$

Eqs. (13) and (14) are employed to determine the distance from ideal and negative ideal alternatives in interval values. In this way we lose less information (data values) than just converting immediately to crisp values.

v. The relative closeness can be calculated as follows:

$$\mathrm{RC}_1 = \frac{D^-_{i2}}{D^+_{i2} + D^-_{i2}}, \quad \mathrm{RC}_2 = \frac{D^-_{i1}}{D^+_{i1} + D^-_{i1}} \tag{15}$$

The final values of RC^*_i are determined as:

$$\mathrm{RC}^*_i = \frac{\mathrm{RC}_1 + \mathrm{RC}_2}{2} \tag{16}$$

As a summary, the Interval-valued fuzzy TOPSIS can be summed up as follows:

- Normalize fuzzy decision matrix $\tilde{X} = [\tilde{x}_{ij}]_{n\times m}$ by Eqs. (10) and (11).
- Determine the ideal solution and the negative ideal solution by Eqs. (12).
- Calculate normalize Euclidean distances by Eqs. (13) and (14).
- Compute the fuzzy relative closeness of each alternative by using each pair of separations (15).

Table 3
The importance of criterion

	DM1	DM2	DM3
C_1	VH	VH	VH
C_2	H	H	MH
C_3	H	MH	MH
C_4	VH	H	VH
C_5	M	MH	M

Table 4
Decision makers' assessments based on each criterion

Decision makers	Volunteer	Criterion		
		DM3	DM2	DM1
C_1	A_1	MG	G	VG
	A_2	MG	G	G
	A_3	VG	G	VG
	A_4	MG	VG	G
C_2	A_1	F	MG	VG
	A_2	MG	VG	VG
	A_3	VG	G	MG
	A_4	VG	F	F
C_3	A_1	G	G	VG
	A_2	G	VG	VG
	A_3	VG	MG	G
	A_4	MG	MG	F
C_4	A_1	VG	G	VG
	A_2	MG	VG	VG
	A_3	VG	VG	G
	A_4	VG	F	G
C_5	A_1	VG	VG	VG
	A_2	G	MG	MG
	A_3	MG	G	G
	A_4	F	G	MG

- Determine the relative closeness by Eq. (16).
- Rank alternatives in terms of their relative closeness's.

5. The application of the extended method in solving problems

Suppose that a Telecommunication Company intends to choose a manager for R&D department from four volunteers named A_1, A_2, A_3, A_4. The decision making committee assesses the four concerned volunteers based on five criteria which follow:

a) proficiency in identifying research areas (C_1),
b) proficiency in administration (C_2),
c) personality (C_3),
d) past experience (C_4) and
e) self-confidence (C_5)

The number of the committee members is three, labeled as DM_1, DM_2, DM_3 respectively.

Each decision maker has presented his assessment based on linguistic variable for rating performance and importance of each criterion by a linguistic variable as depicted in Tables 3 and 4, respectively.

We will proceed to solve the problem using the interval-valued fuzzy TOPSIS. Table 5 shows the final judgment of the decision makers through applying Eq. (9).

Afterwards the decision matrix is normalized by using Eq. (10). Table 6 depicts the normalized decision matrix.

Table 7 shows the weighted normalized matrix. Euclidean distance from the ideal and negative ideal alternatives are calculated using (13) and (14) formulas, respectively. The results have been depicted in Table 8. As demonstrated, the distance from ideal and negative ideal alternatives are determined as an interval.

Applying Eq. (15), the interval relative closeness obtained and the results are depicted in Table 9.

Table 5
The interval-valued fuzzy decision matrix and weights

	C_1	C_2	C_3	C_4	C_5
A_1	[(6.17,7.5),8.67,(9.17,9.83)]	[(5.17,6.17);7.33;(8.17,9)]	[(6.5,8.17);9;(9.33,9.83)]	[(7.5,8.83);9.67;(9.83,10)]	[(8.5,9.5);10;(10,10)]
A_2	[(5.17,6.83),8.33,(9,9.83)]	[(7.17,8.17);9;(9.33,9.83)]	[(7.5,8.83);9.67;(9.83,10)]	[(7.17,8.17);9;(9.33,9.83)]	[(4.83,6.17);7.67;(8.5,9.67)]
A_3	[(7.5,8.83);9.67;(9.83,10)]	[(6.17,7.5);8.67;(9.17,9.83)]	[(6.17,7.5);8.67;(9.17,9.83)]	[(7.5,8.83);9.67;(9.83,10)]	[(5.17,6.83);8.33;(9,9.83)]
A_4	[(6.17,7.5);8.67;(9.17,9.83)]	[(4.5,5.5);6.67;(7.67,8.33)]	[(3.83,4.83);6.33;(7.5,8.83)]	[(5.5,6.83);8;(8.67,9.17)]	[(4.17,5.5);7;(8,9)]
Weight	[(0.85,0.95);1;(1,1)]	[(0.52,0.68);0.83;(0.9,0.98)]	[(0.48,0.62);0.77;(0.85,0.97)]	[(0.75,0.88);0.97;(0.98,1)]	[(0.32,0.42);0.57;(0.7,0.82)]

Table 6
Normalized decision matrix

	C_1	C_2	C_3	C_4	C_5
A_1	[(0.62,0.75);0.87;(0.92,0.98)]	[(0.52,0.62);0.73;(0.82,0.9)]	[(0.65,0.82);0.93;(0.97,1)]	[(0.75,0.88);0.97;(0.98,1)]	[(0.85,0.95);1;(1,1)]
A_2	[(0.52,0.68);0.83;(0.9,0.98)]	[(0.72,0.82);0.9;(0.93,0.98)]	[(0.75,0.88);0.97;(0.98,1)]	[(0.72,0.82);0.9;(0.93,0.98)]	[(0.48,0.62);0.77;(0.85,0.97)]
A_3	[(0.75,0.88);0.97;(0.98,1)]	[(0.62,0.75);0.87;(0.92,0.98)]	[(0.62,0.75);0.87;(0.92,0.98)]	[(0.75,0.88);0.97;(0.98,1)]	[(0.52,0.68);0.83;(0.9,0.98)]
A_4	[(0.62,0.75);0.87;(0.92,0.98)]	[(0.45,0.55);0.67;(0.77,0.83)]	[(0.38,0.48);0.63;(0.75,0.88)]	[(0.55,0.68);0.8;(0.87,0.92)]	[(0.42,0.55);0.7;(0.8,0.9)]

Table 7
Weighted normalized decision matrix

	C_1	C_2	C_3	C_4	C_5
A_1	[(0.52,0.71);0.87;(0.92,0.98)]	[(0.27,0.42);0.61;(0.74,0.89)]	[(0.31,0.5);0.72;(0.82,0.97)]	[(0.56,0.78);0.93;(0.97,1)]	[(0.27,0.4);0.57;(0.7,0.82)]
A_2	[(0.44,0.65);0.83;(0.9,0.98)]	[(0.37,0.56);0.75;(0.84,0.97)]	[(0.36,0.54);0.74;(0.84,0.97)]	[(0.54,0.72);0.87;(0.92,0.98)]	[(0.15,0.26);0.43;(0.6,0.79)]
A_3	[(0.64,0.84);0.97;(0.98,1)]	[(0.32,0.51);0.72;(0.83,0.97)]	[(0.3,0.46);0.66;(0.78,0.95)]	[(0.56,0.78);0.93;(0.97,1)]	[(0.16,0.28);0.47;(0.63,0.8)]
A_4	[(0.52,0.71);0.87;(0.92,0.98)]	[(0.23,0.38);0.56;(0.69,0.82)]	[(0.19,0.3);0.49;(0.64,0.85)]	[(0.41,0.6);0.77;(0.85,0.92)]	[(0.13,0.23);0.4;(0.56,0.74)]

Table 8
The distance from the ideal solution and negative-ideal solution

	$[D_{i1}^{+}, D_{i2}^{+}]$	$[D_{i2}^{-}, D_{i1}^{-}]$
A_1	[1.56,1.96]	[3.6,3.62]
A_2	[1.63,2.01]	[3.54,3.6]
A_3	[1.51,1.92]	[3.66,3.7]
A_4	[2.11,2.37]	[3.05,3.2]

Table 9
The interval of relative closeness

A_1	[0.65,0.7]
A_2	[0.64,0.68]
A_3	[0.66,0.71]
A_4	[0.57,0.59]

Finally, with using Eq. (16), the value of each alternative for final ranking will be:

$$\mathrm{RC}_1^* = 0.673$$
$$\mathrm{RC}_2^* = 0.664$$
$$\mathrm{RC}_3^* = 0.683$$
$$\mathrm{RC}_4^* = 0.583$$

Therefore, the final ranking is:

$$A_3 > A_1 > A_2 > A_4$$

6. Conclusion

It is argued that if a fuzzy MCDM problem is defuzzified into a crisp one in initial steps, then the advantage of collecting fuzzy data becomes unapparent. Based on this fact, we have developed a fuzzy TOPSIS method for dealing with problems, in which criteria values' are interval-valued fuzzy numbers. The proposed fuzzy TOPSIS method combines the TOPSIS method for crisp MCDM with the fuzzy extension principles and performs defuzzification in the final step of decision analysis process. The other aggregation functions can be used to pool the fuzzy ratings of decision-makers in the proposed method. Utilizing the proposed IVF-TOPSIS method, a manager selection problem was examined and the results are demonstrated.

References

[1] C.L. Hwang, K. Yoon, Multiple Attributes Decision Making Methods and Applications, Springer, Berlin Heidelberg, 1981.
[2] K. Yoon, System selection by multiple attribute decision making, Ph.D. dissertations, Kansas State University, Manhattan, Kansas, 1980.
[3] M.A. Abo-Sinna, A.H. Amer, Extensions of TOPSIS for multi objective large-scale nonlinear programming problems, Applied Mathematics and Computation 162 (2005) 243–256.
[4] V.P. Agrawal, V. Kohli, S. Gupta, Computer aided robot selection: the multiple attribute decision making approach, International Journal of Production Research 29 (1991) 1629–1644.
[5] M.F. Chen, G.H. Tzeng, Combining grey relation and TOPSIS concepts for selecting an expatriate host country, Mathematical and Computer Modeling 40 (2004) 1473–1490.
[6] C.T. Chen, Extensions of the TOPSIS for group decision-making under fuzzy environment, Fuzzy Sets and Systems 114 (2000) 1–9.
[7] S.H. Tsaur, T.Y. Chang, C.H. Yen, The evaluation of airline service quality by fuzzy MCDM, Tourism Management 23 (2002) 107–115.
[8] T.C. Chu, Y.C. Lin, A fuzzy TOPSIS method for robot selection, The International Journal of Advanced Manufacturing Technology 21 (2003) 284–290.
[9] E. Triantphyllou, C.T. Lin, Development and evaluation of five fuzzy multi attribute decision making methods, International Journal of Approximate Reasoning 14 (1996) 281–310.
[10] Y.M. Wang, T.M.S. Elhag, Fuzzy TOPSIS method based on alpha level sets with an application to bridge risk assessment, Expert Systems with Applications 31 (2005) 309–319.
[11] L. Zadeh, The concept of a linguistic variable and its application to approximate reasoning - I, Information Science 8 (1975) 199–249.
[12] R. Sambuc, Fonctions ϕ-floues. Application à l'aide au diagnostic en pathologie thyroidienne, Ph.D. Thesis, University of Marseille 1975.
[13] I. Grattan-Guinness, Fuzzy membership mapped onto interval and many-valued quantities, Z. Math. Logik Grundlag. Mathe. 22 (1975) 149–160.
[14] M.B. Gorzalczany, A method of inference in approximate reasoning based on interval-valued fuzzy sets, Fuzzy Sets and Systems 21 (1987) 1–17.
[15] I.B. Turksen, Interval-valued strict preference with Zadeh triples, Fuzzy Sets and Systems 78 (1996) 183–195.
[16] C. Cornelis, G. Deschrijver, E.E. Kerre, Advances and challenges in interval-valued fuzzy logic, Fuzzy Sets and Systems 157 (2006) 622–627.
[17] N.N. Karnik, J.M. Mendel, Operations on type-2 fuzzy sets, Fuzzy Sets and Systems 122 (2001) 327–348.
[18] G. Wang, X. Li, The applications of interval-valued fuzzy numbers and interval-distribution numbers, Fuzzy Sets and Systems 98 (1998) 331–335.
[19] L.J. Kohout, W. Bandler, Fuzzy interval inference utilizing the checklist paradigm and BK-relational products, in: R.B. Kearfort, et al. (Eds.), Applications of Interval Computations, Kluwer, Dordrecht, 1996, pp. 291–335.
[20] H. Bustince, Conjuntos Intuicionistas e Intervalo valorados Difusos: Propiedades y Construcci_on, Relaciones Intuicionistas Fuzzy, Thesis, Universidad P_ublica de Navarra, 1994.
[21] I.B. Turksen, Interval-valued fuzzy sets based on normal forms, Fuzzy Sets and Systems 20 (1986) 191–210.
[22] I.B. Turksen, Interval-valued fuzzy sets and compensatory, Fuzzy Sets and Systems 51 (1992) 295–307.
[23] D.H. Hong, S. Lee, Some algebraic properties and a distance measure for interval-valued fuzzy numbers, Information Sciences 148 (2002) 1–10.
[24] P. Grzegorzewski, Distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on the Hausdorff metric, Fuzzy Set and Systems 148 (2004) 319–328.