A Dual Model for Describing of Reverse Logistics Inventory Systems

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Abstract—**Modeling and analysis of inventory systems in reverse logistics is more complex than in forwards logistics, because in reverse logistics not only amount of demand is not clear, but also uncertainty of product return is appeared in the system. In this paper, an inventory system with the possibility of product return is modeled by means of simulation tools and then thermal equivalent of inventory model, using laws of heat transfer is developed. In order to provide the thermal equivalent, components of the inventory system are known and for each component the thermal equivalent is introduced and then sensitivity analysis is used to show validity of dual model with induction. Hence, analyzing the thermal model can lead us to know the effect of different policies and parameters on inventory system performance. Thermal equivalent model presented in this paper is a strong base for inventory system analysis with more complex structures in future studies.**

Index Terms—**reverse logistics, inventory model, heat transfer, return of products.**

I. Introduction

Enforced legislation and customer expectations increasingly force manufacturers to take back their products after use [6]. This subject is related to the concept of reverse logistics. Reverse logistics (RL) is defined as: the process of planning, implementing and controlling the efficient, cost-effective flow of raw materials, in process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing or creating value or for proper disposal [14]. Inventory management is one of the main process in RL. The focus of this paper is inventory control.

Mathematical inventory models can be divided into deterministic and stochastic models.

In deterministic inventory models information on all the components of the inventory system is assumed to be known with certainty. In particular, demands and returns are known a priori for the entire planning horizon [4]. Several authors have proposed deterministic inventory model. Schrady [15] is believed to be the first to investigate the EOQ model in production/procurement and recovery contexts that disposal is not allowed. Richter [13] developed an EOQ model with constant disposal and used product collection rates.

Several researchers have developed models along the same lines as Schrady and Richter, but with different assumptions, for example, Mabini and Pintelon [12] ,Teunter [17] , Dobos and Richter [3], Koh et al. [11] and Ahmed et al.[1].

Although deterministic models have been widely considered by academics, there are some uncertainties in real world as demand rate, return rate and lead time. Several authors have proposed inventory models taking some uncertainties into account. Heyman [7] considers a continuous review model with independent stochastic demand and returns. Assuming instantaneous procurement and repair and disregarding fixed costs. Fleischmann et al. [6] have presented an inventory model with stochastic demand and return with real assumption. Tang and Grubbstrom [16] have studied a manufacturing/remanufacturing system with stochastic lead times and a constant demand and return.

Applying equivalent systems in many fields of science could facilitate analysis, develop solution and make understanding of complexities. Few researchers in the field of inventory management have used approaches of other science in developing and analyzing inventory models. Jaber and Rosen [9] have applied the first and second laws of thermodynamics to reduce entropy of production system. Theses two researchers developed their model with considering repair and waste disposal [10].

This paper differs from the previous research, since in this study an inventory system in reverse logistics is considered and then thermal equivalent of each part of it is presented, and finally it is shown that the behavior of the two system is like each other, i.d by analyzing the thermal system one can find out behaviors of the inventory system.

The remainder of this paper is organized as follows. Section 1 describes inventory system framework. In section 2, simulation model of inventory system is developed. In section 3, we have presented thermal equivalent of inventory system. Section 4 presents validity of simulation and thermal model and in section 5, summary and conclusions are given.

A. *Definition of inventory system*

The inventory system studied in this paper like other studies [5] consists of two warehouses: warehouse 'a' where the returned product 'A' is stored, and warehouse 'A' where the product 'A', is to be supplied for the market, is stored. Figure 1 shows the structure of this system.

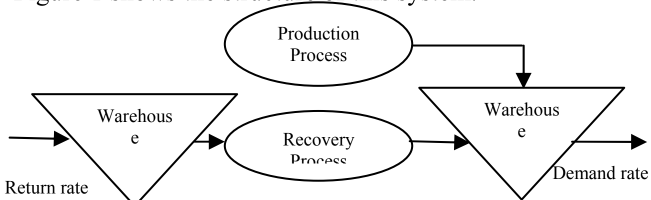

Figure 1- Structure of inventory system

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As is shown in Figure 1, required products from the warehouse 'A' to meet the demand rate is presented in two ways: one way is to recover the returned products of the warehouse 'a' into product 'A', another way is direct production of product 'A'. In order to model the above system, following assumptions are considered:

- Return rate and demand rate have a Poisson distribution.
- Return rate and demand rate are independent.
- Production and recovery Lead times are deterministic.
- Production and recovery rate in time unit always is greater than rate of demand in time unit.
- Disposal is not taken in to account.
- Shortage is included as backlog.

Notations

Notations of thermal model	Notations of simulation model
u_a : Heat production rate inside body 'a' at each time unit.	λ_a : Return rate of products at each time unit.
$delaytime$: Duration of heat production inside body 'A'.	μ_{aA}, μ_A : Production and recovery rate at each time unit.
$T_{\inf\_A}$: Temperature of environment of body 'A'.	λ_A : Demand rate at each time unit.
T_{SA} : Threshold temperature of body 'A'.	S_A : Reorder point.
u_A : Heat production rate inside body 'A' in time unit.	Q_A : Economic level of production.
K_{aA} : Thermal conductivity of body 'A'.	R_a : Economic level of recovery.
T_a : Temperature of body 'a'.	$\overline{a}$: Mean inventory of returned products in warehouse 'a'.
T_A : Temperature of body 'A'.	$\overline{A}$: Mean inventory of products 'A' in warehouse 'A'.
n_A : Number of times element turns on inside body 'A'.	$\overline{A\_}$: Mean shortage of products 'A'.
n_{aA} : Number of transferred energy bundles from body 'a' to body 'A'.	N_A : Number of production times in planning horizon.
γ_a, γ_A : Specific heat capacity of body 'A' and body 'a'.	N_{aA} : Number of recovery times in planning horizon.
	r : Transforming coefficient of returned products into products 'A'.
	T : Planning horizon.

B. Inventory policy

When warehouse 'A' reaches the level of S_A, then we have two situations:

a) If inventory level in warehouse 'a' has reached the economic level R_a, then to the amount of R_a products units enter to warehouse 'A' during recovery process.

b) If inventory level in warehouse 'a' is less than R_a, then to the amount of Q_A product units will enter to warehouse 'A' during production process.

II. Presentation of Simulation Model

The method of developing simulation model is systematic. It means that firstly, subsystems are designed separately then, by some tools, the interactions between subsystems are defined, according to figure 1 main subsystems are: warehouse 'a', warehouse 'A', and production subsystem of product 'A'. In addition to this, main interactions are: interaction of warehouse 'a' with warehouse 'A', and interaction between production subsystem of product 'A' and warehouse 'A'. To design subsystems and interactions between them, a computer program is designed and by programming language of Visual Basic is prepared that in the following, the way of receiving inputs and providing outputs in the program are described.

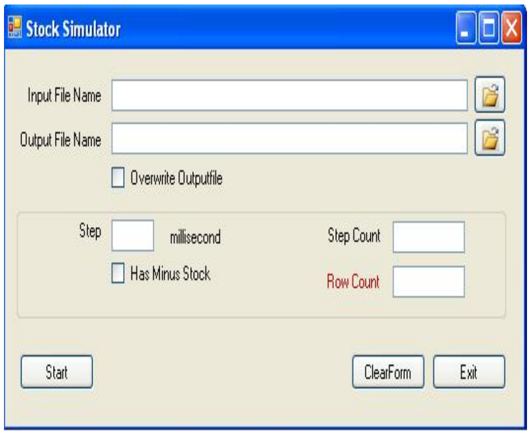

Figure 2- Data input form in simulation program

Simulation program receives the input parameters as an Excel file and in the end it stores and provides the output in the Excel file. Figure 2 shows data input form in simulation program.

In this form in the section 'input File Name', the path of input parameters file is introduced to the program and in the section 'output File Name', storage place of the Excel file output is introduced. 'Step' indicates simulation steps that at each step, information of warehouses are saved, and 'Step Count' is finish point of the simulation.

Input parameters for carrying out simulation are: λ_a, μ_{aA}, μ_A, λ_A, S_A, Q_A, R_a and r. Value of these parameters on each line of Excel input file is respectively inserted and for each line, simulation one time is carried out and the results in Excel output file will be inserted. In other words, in this way by changing the values of a parameter on different lines, one can see the effect of it on the outputs under study.

$\overline{a}$, $\overline{A}$, $\overline{A\_}$, N_A and N_{aA} are simulation outputs for each run, which is stored in Excel output file and can be analyzed. These outputs are importance, because main costs of inventories depend on mean inventory of warehouses and number of set-up times and fixed costs of providing products through production or recovery process.

A. Presentation of thermal model

In order to present the thermal model, for each part in inventory model the logical element is defined in the thermal mode. In the following, each component of the inventory model together with its thermal equivalent is described.

Warehouses: In the thermal model, Warehouses are two bodies which in addition to the environment; they have thermal exchange with each other. Inventory of the product

'A' is considered in thermal model of temperature of the body 'A', which changes as a result of thermal exchange with body 'a' and with the environment. Inventory of the returned product 'A', in the thermal model is considered equivalent to the temperature of the body 'a', Figure 3 shows the bodies next each other and fleshes in this figure indicate the direction of thermal exchange.

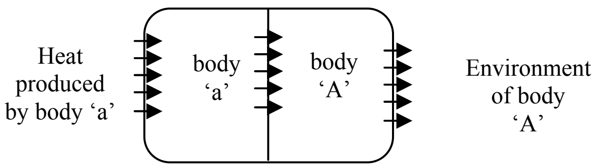

Figure 3- Structure of thermal model

Rate of demand and return: As is shown in figure 3, each body is exchanging heat with its environment.

Environment of body 'A' is cooler than body 'A'; hence the heat always flows from body 'A' toward the environment. In other words, the cooler the environment is, the greater the rate of heat transfer is and the more decreases the temper true of body 'A'. Temperature of body A's environment is considered equivalent to demand rate of body 'A'.

Return rate is equivalent of the heat produced inside body 'a'. As is shown in figure 3, direction of the fleshes is toward inside body 'a' indicating entering of heat into body 'a'.

Reorder point: When inventory of 'A' reaches S_A level it expresses need and at this point it is necessary its need to be satisfied through recovery of the returned products or production of product 'A'.

In thermal model, threshold temperature T_{SA} for body 'A' has been considered which needs to be always greater than threshold temperature. If temperature of body 'A' reaches a temperature below or equal to threshold temperature, internal element of body 'A' turns on and produces a heat equivalent of u_A which corresponds to the production rate in simulation model.

Economic level of recovery: If warehouse of the returned products reaches the level R_a it is ready for recovery process and recovery process will be economic. For equivalent definition of this variable, Thermal conductivity of body 'a' (K_{aA}) is used. The greater Thermal conductivity is, the better the heat transfer is being done. In other words, one might say that Thermal conductivity has reversed relationship with variable R_a. A high Thermal conductivity means transfer has been easily done and in the time simulation model greater amount of transfer takes place, in order to choose a smaller amount for R_a.

Transforming coefficient: Transforming coefficient means that for each returned product how many products 'A' will be reached after recovery process. Specific heat is considered as equivalent it.

Economic level of production: when inventory level of product 'A' reaches the threshold level of S_A, but it is not possible to meet this need by recovering the returned products, this need will be met by direct production of product 'A'. In order to determine equivalent amount of production in thermal model, it is assumed that there is an element inside body 'A' which, when the temperature reaches the threshold level T_{SA}, turns on for the duration of $delay\ Time$ and at each time unit gives a heat of u_A to body 'A'. Thus, equivalent of Q_A in the thermal model is: $u_A \times delay\,Time$.

Number of times of production and recovery in time horizon: number of times of production in the simulation model is an equivalent of number of times that the element turns on inside body 'A'. But since the heat transfer from body 'a' to body 'A' is a continuous process, in order to calculate the effect of body 'a' on the temperature of body 'A', the amount of heat transfer by the element each time that it turns on is considered and based on this, number of bundles of heat transfer is calculated. Equations 1 and 2 clearly show how to calculate number of transferred bundles.

$$Q_A = u_A \times delay\,Time \tag{1}$$

$$n_{aA} = \frac{Q_{aA}}{Q_A} \tag{2}$$

where Q_{aA} is total amount of heat transfer from body 'a' to body 'A' in time horizon.

III. Differential Equations

Thermal energy entering into and coming out of bodies 'a' and 'A' changes their temperature. In this section, in order to study heat changes of bodies, their differential equations are presented.

As is shown in figure 3, body 'a' receives heat from internal element and gives heat to body 'A'. Received heat from environment is like the element which turns on inside body 'a' and always gives it a heat of u_a at each time unit.

Heat transfer from body 'a' to body 'A' is modeled based on the Fourier's law. According to this law heat transfer from body 'a' to body 'A' has direct relation with Thermal conductivity of body 'a' (K_{aA}), temperature difference of the two bodies (ΔT), and interface of the two bodies (A), and has reverse relation with distance of the two bodies (ΔL). Equation 3 shows the Fourier's law (Incropera & Dewitt, 2001).

$$q = kA\frac{\Delta T}{\Delta L} \tag{3}$$

Based on the above explanation, differential equation which shows temperature changes of body 'a' is:

$$\frac{K_{aA}A}{L_{aA}}(T_A - T_a)H(T_a - T_A) + u_a = \gamma_a \frac{dT_a}{dt} \tag{4}$$

Left side of the equation 4 shows thermal energy entering into or coming out of body 'a', and right side of this equation shows temperature changes of body 'a'. In this equation $H(T_a - T_A)$, is heaviside function which is used to define unilateral heat exchange from body 'a' to body 'A' as cooler body. Expression 5 shows how this function works.

$$H(x) = \begin{cases} 1 & x > 0 \\ 0 & x \le 0 \end{cases} \tag{5}$$

Body 'A' receives heat from body 'a' and also receives heat whenever the internal element turns on. Heat transfer from body 'A' to environment also occurs in a continuous process because temperature of environment is always lower than that of body ‘A’. Differential equation 6 shows heat changes of body ‘A’.

$$\frac{K_{aA}A}{L_{aA}}(T_a - T_A)H(T_a - T_A) + H(T_{SA} - T_A)u_A + H(T_A - T_{\inf\_A}).h_A(T_{\inf\_A} - T_A) = \gamma_A \frac{dT_A}{dt} \qquad (6)$$

where the term $h_A(T_{\inf\_A} - T_A)$ shows heat exchange between environment and body ‘A’. Since in this case heat transfer is between body and environment, hence the above equation which is known as Newton’s law of cooling is used (Increopera & Dewitt, 2001).

Solution of thermal model

To solve Differential equations 5 and 6 numerical method of Runge-Kutta is used. This method was developed in 1900 by two German mathematicians, C.Runge and M.W.Kutta. In the following, the Runge-Kutta’s method of forth order is explained (Braun, 19910). Consider the Differential equation $y' = f(t, y)$ with initial value of $y(t_0) = y_0$. Based on the Runge-Kutta of forth order we have:

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$t_{n+1} = t_n + h$$

where h is the step of each phase of numerical solution. Values of k are:

$$k_1 = hf(t_n, y_n) \qquad k_3 = hf(t_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2)$$

$$k_2 = hf(t_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1) \qquad k_4 = hf(t_n + h, y_n + k_3)$$

IV. Validity Analysis

In this section, this question is answered: is behavior of thermal model similar to simulation model or not? For validity analysis a numerical example is provided and for different values of parameters, sensitivity analysis is applied. In table 1 different value of parameters and variables of simulation and thermal model are provided.

TABLE 1- Numerical Example Data

Parameters of simulation model	value	Parameters of thermal model	value
R_a	61	$K_{aA}A$	2.5
S_A	35	T_{SA}	10
λ_a	3	u_a	70
λ_A	10	$T_{\inf\_A}$	-10
μ_{aA}, μ_A	200	$delay\,Time$	0.5
Q_A	49	u_A	30
r	1	γ_a, γ_A	1

Parameters S_A in simulation model and T_{SA} in thermal model are equivalent. In simulation model, S_A was the threshold level which in case of inventory reduction to the level of S_A, has to be refilled, and T_{SA} is the threshold level of temperature which in case of temperature reduction to the level of T_{SA}, it is necessary that the mechanism of increasing temperature to be activate.

It is expected that with increase of threshold level of inventory, mean inventory of warehouses to increase too and equivalently with increase of threshold temperature mean temperature to increase too. Diagram 4 shows changes of inventory level of warehouse ‘A’ and equivalently temperature of body ‘A’ relative to changes of S_A and T_{SA}. In addition to, t-student test for measuring equality of behavior two systems have been done (Table 2). As is shown in diagram 4, with increase of S_A and T_{SA}, mean amounts of inventory in warehouse ‘A’ and mean temperature of body ‘A’ has increased with an almost linear trend.

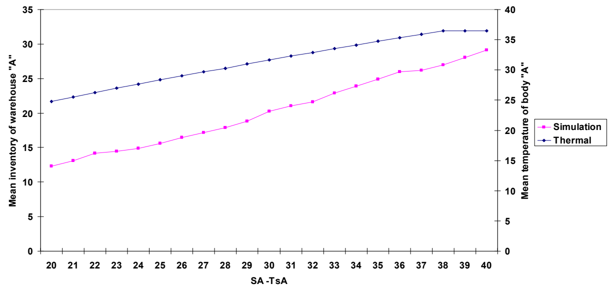

Figure 4- Effect of threshold levels on mean inventory of warehouse “A” and temperature of body “A”.

TABLE 2- Test for equality of means between inventory of warehouse “A” and temperature of body “A”

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
VAR00006	Equal variances assumed	17.159	.000	.510	40	.613	1.03964	2.03656	-3.07640	5.15567
	Equal variances not assumed			.510	27.911	.614	1.03964	2.03656	-3.13266	5.21194

The heat which is produced with constant rate inside body ‘a’ is equivalent of return rate of product ‘A’. It is expected that with increase of return rate of product ‘A” mean inventory level in warehouse “a” to increase and equivalently with increase of the heat produced in body ‘a’, mean temperature of this body to increase too. Diagram 5 shows changes of mean inventory of warehouse ‘a’ and mean temperature of body ‘a’ relative to changes of return rate and heat produced in body ‘a’. As could be seen from diagram 5 by changes in the two equivalent parameters, the two systems have ascending behavior and produce like each other. In addition to, t-student test for measuring equality of behavior two systems have been done (Table 3)

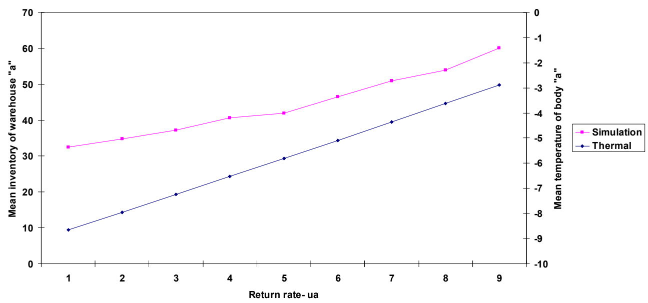

Figure 5- Effect of return rate and u_a on mean inventory of warehouse “a” and temperature of body “a”.

It is likely with increase of demand rate for product ‘A’, inventory level in warehouse ‘A’ to decrease, because in this case exit of products from warehouse ‘A’ takes place more intensely. In thermal model, the environment was a factor which caused temperature decrease in body ‘A’. In other words, in this model the cold is considered as and equivalent to customer; the cooler the environment is, the lower mean temperature of body ‘A’ is expected to be. In diagram 6, horizontal axis represents changes in demand rate and absolute value of environmental temperature of body ‘A’ which is calculated for different levels of these two parameters. As could be seen from diagram 6, with increase of demand rate and absolute value of environmental temperature of body ‘A’, inventory level and temperature of body ‘A’ decrease. Reduction in temperature of body ‘A’ occurs for the reason that by cooling down of the environment, heat transfer from body “A” to the environment takes place faster and temperature of body ‘A’ decreases. In addition to, t-student test for measuring equality of behavior two systems have been done (Table 4).

TABLE 3- TEST FOR EQUALITY OF MEANS BETWEEN INVENTORY OF WAREHOUSE “A” AND TEMPERATURE OF BODY “A”

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference Lower	Upper
VAR00003	Equal variances assumed	4.364	.053	-1.360	16	.193	-9.28709	6.82789	-23.76156	5.18739
	Equal variances not assumed			-1.360	11.885	.199	-9.28709	6.82789	-24.17981	5.60564

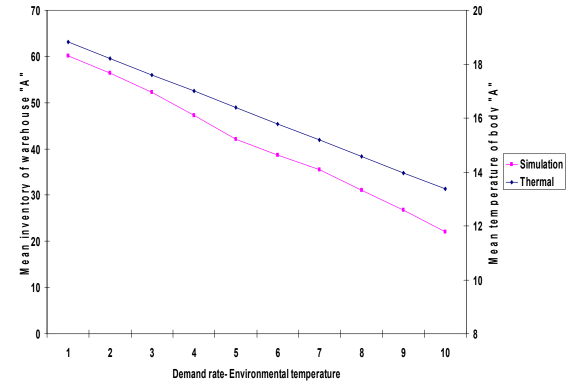

Figure 6- Figure 6- Effect of demand rate and environmental temperature on mean inventory of warehouse “A” and temperature of body “A”.

TABLE 4- TEST FOR EQUALITY OF MEANS BETWEEN INVENTORY OF WAREHOUSE “A” AND TEMPERATURE OF BODY “A” (RELATED TO FIGURE 6)

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference Lower	Upper
VAR00007	Equal variances assumed	9.188	.007	-.233	18	.818	-.99948	4.28362	-9.99904	8.00008
	Equal variances not assumed			-.233	11.292	.820	-.99948	4.28362	-10.39802	8.39907

Recovery economic level of returned products is equivalent to Thermal conductivity coefficient of body ‘A’. With increase of R_a, number of times of satisfying needs of warehouse ‘A’ in planning horizon decreases through recovery process, because in this case, waiting time of warehouse ’a’ till reaching the level of R_a will increase. With increase of Thermal conductivity coefficient, the process of heat transfer between bodies ‘a’ and ‘A’ faster will take place. Therefore the two parameters act in opposite directions. To calculate the amount of heat transferred from body ‘a’ to body ‘A’, it is assumed that the heat as bundles of Q_A will be transferred which Q_A is equal to the amount of heat produced each time by the element inside body “A” being turned on. Thus, the amount of Q_A is: $Q_A = u_A \times delayTime$.

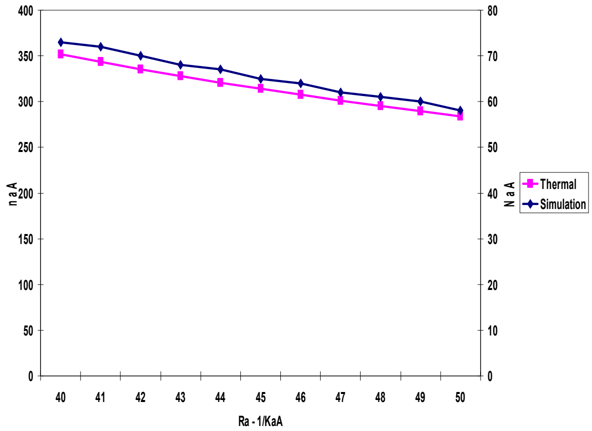

Figure 7- Effect of R_a and $1/K_{aA}$ on number of production times in planning horizon and number of transferred energy bundles from body ‘a’ to body ‘A’.

TABLE 5- TEST FOR EQUALITY OF MEANS BETWEEN NUMBER OF PRODUCTION TIMES IN PLANNING HORIZON AND NUMBER OF TRANSFERRED ENERGY BUNDLES FROM BODY ‘A’ TO BODY ‘A’.

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference Lower	Upper
VAR00001	Equal variances assumed	.062	.806	-.134	20	.895	-1.32893	9.91332	-22.00776	19.34991
	Equal variances not assumed			-.134	19.928	.895	-1.32893	9.91332	-22.01257	19.35472

In diagram 7, horizontal axis represents the two parameters R_a and $\frac{1}{K_{aA}}$ and vertical axes shows number of times of meeting the needs of warehouse ‘A’ through recovery of the returned products ‘A’ and number of heat bundles transferred from body ‘a’ to body ‘A’ for $T = 1000$ .As one can see, with increase of values of horizontal axis, number of meeting times and equivalently number of transferred heat bundles decrease. In addition to, t-student test for measuring equality of behavior two systems have been done (Table 5).

If the warehouse of the returned products ‘A’ does not reach the economic level recovery and the warehouse ‘A’ doesn’t reach the level of S_A, it is necessary that the need of the warehouse ‘A’ to be met by production of products ‘A’. It is expected that with increase of production volume at each time unit, mean inventory of warehouse ‘A’ to increase and equivalently with increase of heat amount produced (in time unit) by the element inside body ‘A’, mean temperature of body ‘A’ to increase too. Diagram 8 shows mean changes of inventory in warehouse ‘A’ relative to changes in production level and the heat produced by the element. As is shown in figure 8, with increase of the two equivalent parameters of economic level of production and the heat produced, amount of the two equivalent variables of mean inventory and mean temperature of body ‘A’ increase as well. In addition to, t-student test for measuring equality of behavior two systems have been done (Table 6).

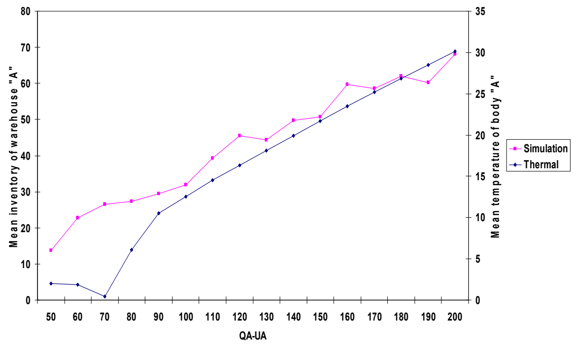

Figure 8- Effect of Q_A and U_A on mean inventory of warehouse "A" and temperature of body "A".

TABLE 6- TEST FOR EQUALITY OF MEANS INVENTORY OF WAREHOUSE "A" AND TEMPERATURE OF BODY "A". (RELATED TO FIGURE 8)

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
VAR00013	Equal variances assumed	.978	.334	-.065	22	.949	-.36804	5.68085	-12.14940	11.41333
	Equal variances not assumed			-.065	20.979	.949	-.36804	5.68085	-12.18274	11.44667

V. CONCLUSIONS

In this paper, an inventory system in reverse logistics is introduced and then for each component of the above mentioned system, its equivalent parameter in the field of heat transfer is presented. In order to analyze inventory system, a simulation model is developed and by using it for each specific input, evaluation criteria of the system such as mean inventory of warehouse, number of production times and recovery times can be calculated during planning horizon. And for thermal system analysis, system differential equations are determined and then by using numerical method are solved. Results of sensitivity analysis of simulation- and thermal models indicates that simulation and thermal models are equivalents of one another, therefore, by analyzing the thermal model one can reduce complexities of analyzing simulation model and achieve suitable results. For example, variable R_a in simulation model is economic level of recovery of returned product 'A' into product 'A' which in critical conditions (under reduction) produces unstable behavior. Instability means that inventory of products "A" is not able to respond to demand of customers, and inventory becomes more and more negative. In heat transfer model, this phenomenon can be explained with under increase of Thermal conductivity coefficient becomes large enough which corresponds to R_a in simulation model, body 'a' to 'A'. The great Thermal conductivity coefficient becomes, the more conductibility becomes, and heat transfer will be done easier. In body 'a', if Thermal conductivity coefficient becomes large enough which corresponds to R_a in simulation model, body 'a' becomes intensely conductive, i.d. Current from surrounding environment of 'a' to 'A' without interference of 'a' is established, and in fact a kind of short connection is created which directly establishes environment current with 'A'. In inventory model, the preference for meeting needs of warehouse 'A' when it reaches S_A level is recovery of returned products. Therefore, in a condition that meeting process is being carried out through recovery, it's not possible to meet (the need) through production process. Now, if at each time of transfer from 'a' to 'A', small amount of products (R_a) is transferred, inventory of warehouse 'A' will become more and more negative, because boundless of R_a numbers which is transferred from 'a' to 'A' in short times and successively, don't allow (for) production process. So, in a condition that R_a level is chosen lower, it is like direct entering of returned products into recovery process.

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